1 Height up the wall $= \sqrt{18^2 - 7^2}$
$= 5\sqrt{11}$ metres

2 Length of diagonal $= \sqrt{40^2 + 9^2}$
$= 41$ metres

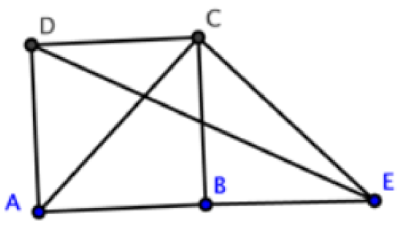
3 Distance of the chord from $O = \sqrt{14^2 - 2^2}$
$= \sqrt{192}$
$= 8\sqrt{3}$ cm

4 Length of diagonal $= \sqrt{13^2 + 13^2}$
$= 13\sqrt{2}$ cm

5 a Let x cm be the length of a side of the square.
$2x^2 = 100$
$x^2 = 50$
$x = \sqrt{50}$
The length of a side is $5\sqrt{2}$ cm. The area $= 50$ cm^2.

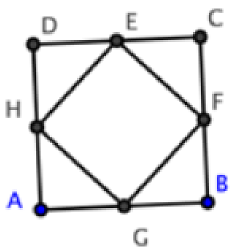
b Let x cm be the length of a side of the square.
$2x^2 = 64$
$x^2 = 32$
$x = 4\sqrt{2}$
The length of a side is $4\sqrt{2}$ cm Area $= 32$cm^2

6



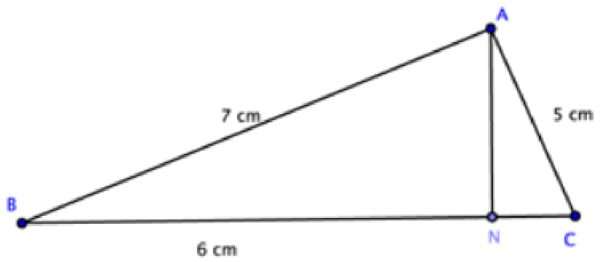
$\triangle ACB \equiv ECB$ (RHS)
$\therefore AB = BE$
Each side length of the square has length $= 2$cm
$\therefore DE^2 = 2^2 + (2 \times 2)^2$
$\therefore DE^2 = 20$
$\therefore DE = 2\sqrt{5}$cm

7



E, F, G and H are the midpoints of sides DC, CB, BA, AD respectively.
$DE = EC = CF = FB = BG = GA = AH = HD = 1$ cm
We see that:
$HE^2 = 1 + 1 = 2$ and therefore
$HE = EF = FB = GA = \sqrt{2}$. $\therefore$ $EFGH$ is a rhombus.
$\triangle HDF \equiv \triangle CEF$ (SAS) These triangles are right-angled isosceles triangles and therefore
$\angle DEH = \angle CEF = 45°$.
Therefore $\angle HED$ is a right angle and $EFGH$ is a square.
The area of $EFGH$ is 2 cm^2

8



Let $CN = x$ cm
Then in $\triangle ABN$
$(6 - x)^2 + AN^2 = 7^2 \ldots (1)$
In $\triangle ACN$
$x^2 + AN^2 = 25 \ldots (2)$
Subtract (2) from (1).
Then

$$-12x + 36 = 49 - 25$$
$$-12x = -12$$
$$x = 1$$

Substitute in (2)

$$1 + AN^2 = 25$$
$$AN = \sqrt{24}$$
$$\therefore AN = 2\sqrt{6} \text{ cm}$$

9 **a** $7^2 \neq 5^2 + 6^2$ (Not three sides of a right-angled triangle)

b $3.9^2 = 3.6^2 + 1.5^2$ (Three sides of a right-angled triangle)

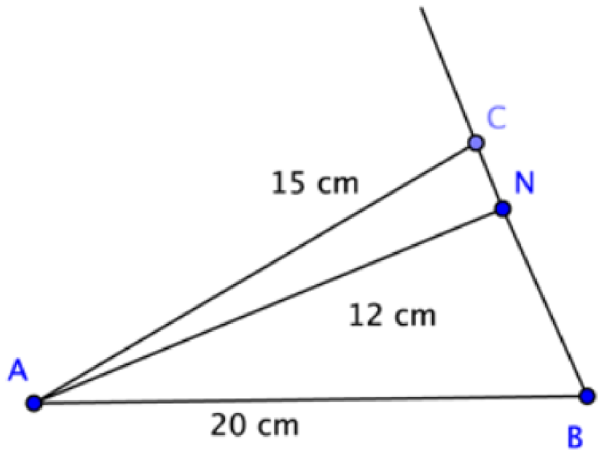
c $4^2 \neq 2.4^2 + 2.4^2$ (Not three sides of a right-angled triangle)

d $82^2 = 18^2 + 80^2$ (Three sides of a right-angled triangle)

10
$$(x^2 - 1)^2 + 4x^2 = x^4 - 2x^2 + 1 + 4x^2$$
$$= x^4 + 2x^2 + 1$$
$$= (x^2 + 1)^2$$

The converse of Pythagoras' theorem gives that the triangle is right-angled.

11



Let $NC = x$ cm

In $\triangle ACN$

$x^2 = 15^2 - 12^2$ Let $NB = y$ cm

$x^2 = 225 - 144$

$x^2 = 81$

$x = 9$

In $\triangle ABN$

$y^2 = 20^2 - 12^2$ $\therefore BC = x + y = 25$

$y^2 = 400 - 144$

$y^2 = 256$

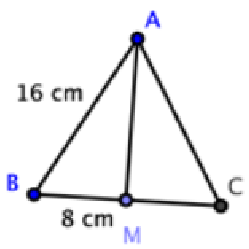
$y = 16$

The sides of $\triangle ABC$ are 20, 15, and 25.

$$\begin{aligned}20^2 + 15^2 &= 400 + 225 \\ &= 625 \\ &= 25^2\end{aligned}$$

The converse of Pythagoras' theorem gives that the triangle is right-angled.

12



$AM^2 = 16^2 - 8^2 = 192$

$\therefore AM = \sqrt{192} = 8\sqrt{3}$

13

$$A_3 = \frac{1}{2}\pi\left(\frac{c}{2}\right)^2$$

$$A_2 = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2$$

$$A_1 = \frac{1}{2}\pi\left(\frac{a}{2}\right)^2$$

Adding A_1 and A_2

$$\frac{1}{2}\pi\left(\frac{b}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{1}{8}\pi(a^2+b^2)$$
$$= \frac{1}{8}\pi^2$$
$$= A_3$$

14 $BD^2 = 8^2 + 6^2 = 100$

$\therefore BD = 10 \ \triangle AXB \equiv \triangle CYD \qquad \text{(ASA)}$

$$\text{Let} BX = DY = x$$
$$\text{In} \triangle AXB$$
$$AX = \sqrt{36 - x^2}$$

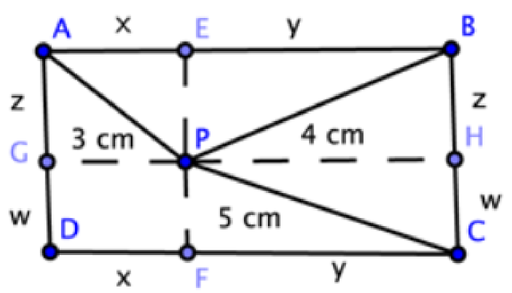
In $\triangle AXD$

$$AX^2 + XD^2 = 64$$
$$36 - x^2 + (10-x)^2 = 64$$
$$36 - x^2 + 100 - 20x + x^2 = 64$$
$$20x = 72$$
$$x = \frac{18}{5}$$
$$\therefore XY = 10 - 2x$$
$$= 2.8$$

15 From the two right-angled triangles

$$36 = (x+4)^2 + y^2 \ldots (1)$$
$$9 = x^2 + y^2 \ldots (2)$$
$$\text{Subtract (2) from (1)}$$
$$27 = x^2 + 8x + 16 - x^2$$
$$27 = 8x + 16$$
$$11 = 8x$$
$$x = \frac{11}{8}$$
$$\text{Substitute in (2)}$$
$$9 = \left(\frac{11}{8}\right)^2 + y^2$$
$$9 - \frac{121}{64} = y^2$$
$$y = \frac{\sqrt{455}}{8}$$

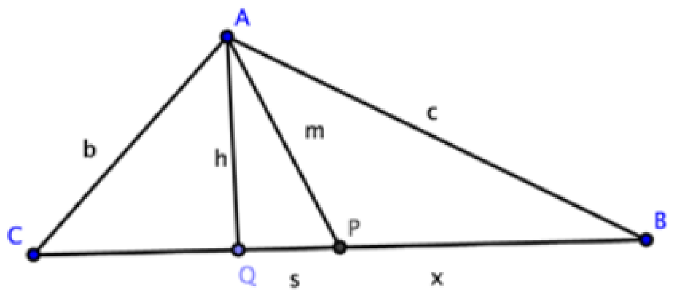
16



$$\text{Let } AE = DF = x$$
$$\text{Let } BE = CF = y$$
$$\text{Let } AG = BF = z$$
$$\text{Let } GD = HC = w$$
$$\text{Using Pythagoras's theorem 3times}$$

$x^2 + z^2 = 9 \ldots (1)$
$y^2 + z^2 = 16 \ldots (2)$
$w^2 + y^2 = 25 \ldots (3)$
Subtract (1) from (2)
$y^2 - x^2 = 7 \ldots (4)$
Subtract (4) from (3)
$w^2 + x^2 = 18$
$PD^2 = w^2 + x^2 = 18$
$PD = 3\sqrt{2}$ cm

17



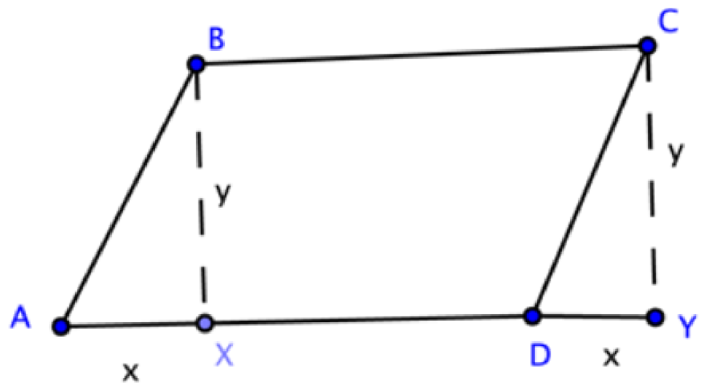
Let $AB = c, AC = b, PB = x,$
$AP = m, AQ = h, CQ = t, QP = s$

Then,

$$s + t = x$$
$$m^2 = h^2 + s^2$$
$$c^2 = h^2 + (s+x)^2$$
$$b^2 = h^2 + t^2$$

We start with,

$$AB^2 + AC^2 - 2AP^2$$
$$= c^2 + b^2 - 2m^2$$
$$= h^2 + (s+x)^2 + h^2 + t^2 - 2h^2 - 2s^2$$
$$= h^2 + s^2 + 2sx + x^2 + h^2 + t^2 - 2h^2 - 2s^2$$
$$= s^2 + 2xs + x^2 + t^2 - 2s^2$$
$$= x^2 + 2xs + t^2 - s^2$$
$$= x^2 + 2xs + (t-s)(t+s)$$
$$= x^2 + 2sx + (t-s)x$$
$$= x^2 + sx + tx$$
$$= x^2 + x(s+t)$$
$$= 2x^2$$
$$= 2PB^2$$
$$\therefore AB^2 + AC^2 - 2AP^2 = 2PB^2$$
$$\therefore AB^2 + AC^2 = 2PB^2 + 2AP^2$$



$$\triangle ABX \equiv \triangle CYD\ (RHS)$$
Let $AX = BY = x$
Let $BX = CY = y$

$$AC^2 = (AD + x)^2 + y^2 \dots (1)$$
$$BD^2 = (AD - x)^2 + y^2 \dots (2)$$

Add (1) and (2)

$$AC^2 + BD^2 = AD^2 + 2xAD + x^2 + AD^2 - 2xAD + y^2$$
$$= 2AD^2 + 2(x^2 + y^2)$$
$$\therefore AC^2 + BD^2 = 2AD^2 + 2AB^2.$$